Impact of Liability Rules on Rear Ending Crashes

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Setting the Stage
Motivation

• Rear-Ending Crashes on Congested Freeways.
• Problems: i) Non-recurring congestion
  ii) Quite Frequent.
• Primary reason: Long reaction time (Davis & Swenson, 2006)
• Inadequate Liability rules (e.g. internalized delay costs)
• Understand driver-to-driver interaction.
Game: Motorist-Pedestrian

- Players: “who” (Motorists and Pedestrian)
- Strategies: “who can do what when”
  - i) Due care. ii) No care.
- Payoffs: “who gets what”
- Information: who knows what when
Assumptions of the Game

i) Accidents bound to happen unless both players exercise care.


iii) Cost of injury (pedestrian): $100.

iv) Prob. of crash=0.10 (both parties exercise care)

Source: “Game Theory And Law” by Baird, Gertner and Picker
Regime of no Liability

• Pay-off matrix as a Normal form

<table>
<thead>
<tr>
<th>PEDESTRIAN</th>
<th>MOTORIST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NO CARE</td>
</tr>
<tr>
<td>NO CARE</td>
<td>(-100, 0)</td>
</tr>
<tr>
<td>DUE CARE</td>
<td>(-110, 0)</td>
</tr>
</tbody>
</table>

• Solution Concept: Nash Equilibrium
Animated Representation of Rear-Ending Event

Triggers

Driver 0

Driver 1

Driver 2
Algebraic Conditions for rear-ending event

- Simplest condition:
  i) Equal headways.
  ii) Vehicles traveling at same speed.

- Available braking distance for driver 2 (Davis & Swenson, 2006)
  \[ d_2 = \frac{v^2}{2a_0} + v(h - r_1) + v(h - r_2) \]

- Condition for collision
  \[ d_2 < \frac{v^2}{2\ddot{a}} = d_c, \text{ where } \ddot{a} = \max \text{ braking} \]
Collision Speed (reaction times)
Pay-offs for Drivers

\[ U_i = U_0 - \lambda (r_i - r_0)^2, \quad \text{if } d_2 > d_c \]

\[ = U_0 - \lambda (r_i - r_0)^2 - \beta \times L_i(a_0, r_1, r_2) \quad i = 1, 2 \]

\[ L_i(a_0, r_1, r_2) = \sqrt{v^2 - 2\hat{a}\left[\frac{v^2}{2a_0} + v(2h - r_1 - r_2)\right]} \]

\( r_0 = \text{Desired reaction time} \)

\( \lambda = \text{Penalty of deviating from desired reaction time} \).

\( \beta = \text{Loss proportionality} \)

\( U_0 = \text{Base utility} \)
Continuous Utility

Driver 1's utility as a function of reaction time

Driver 2's reaction time = 2 secs
Loss prop(\(\beta\)) = 1
\(v = 60 \text{ ft/sec}\)
\(a_0 = 10 \text{ ft/sec}^2\)
Goal: Population of “Safe” Drivers

• Driver state:
  (i) Dangerous (Inattentive)
  (ii) Safe (Attentive)

• Ideal Population: 100% safe drivers.

• Is this ideal population stable?

• Incentive for being inattentive.
Evolutionary Game Concepts

• Population has finite states:
  e.g. Hawk-Dove Game
• Individual’s utility (fitness) depends on the behavior of others.
• Pay-offs are frequency dependent.
  i.e. success of each strategy depends on frequency of its own and other strategies.
• In our model: a) Attentive  b) Inattentive
Evolutionary Stable Strategies

• Fundamental Questions:
  Is any strategy stable against invasion of every other strategy?

• ESS: Strategy X is ESS iff
  (i) $U(X,X) > U(Y,X)$ or
  (ii) $U(X,X) = U(Y,X)$ & $U(X,Y) > U(Y,Y)$

where $U()$ is the utility function
Is all “Safe” Driver: Invincible state

- Attentive state (A): \( r_A = 1 \) sec
- Inattentive state (I): \( r_I = 3 \) sec
- Driver Utility Table

<table>
<thead>
<tr>
<th></th>
<th>Driver 1</th>
<th></th>
<th>Driver 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Driver 1</td>
<td>A</td>
<td>I</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>6,6</td>
<td>6,10</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>10,6</td>
<td>-590.5,-590.5</td>
<td></td>
</tr>
</tbody>
</table>

- \( U(A,A) = 6 < U(I,A) = 10 \)
- Only ESS : A=99.3% ; I=0.7%
Population Dynamics

- Dynamic version of ESS.
- Models change in the frequency of strategies in the population.
- Long term proportion of attentive and distracted drivers.
- Various dynamics in the literature.
- Logit dynamics (details in the paper)

Evolution of Driver Population

\[ \pi_A(t+1) \]

\[ \pi_A(t) \]

A = 94.5%, I = 5.4%
Impact of Liability Rule

- Driver Population:
  (i) A: Reaction time = 1.5 secs.
  (ii) I: Reaction time = 3.0 secs.
- Accident avoided if both drivers are A.
- Compare two liability rule:
  (i) No liability (Loss equally shared).
  (ii) Negligence with Contributory Negligence (NCN)
Comparing Driver Pay-offs Under Different Liability Rules

<table>
<thead>
<tr>
<th></th>
<th>Without Contributory Negligence</th>
<th>With Contributory Negligence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>I</td>
</tr>
<tr>
<td>A</td>
<td>7.75</td>
<td>-10.5</td>
</tr>
<tr>
<td>I</td>
<td>-8.31</td>
<td>-49.74</td>
</tr>
</tbody>
</table>
Comparing Stationary Distribution

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>With NCN</td>
<td>87%</td>
<td>13%</td>
</tr>
<tr>
<td>No Liability</td>
<td>97%</td>
<td>3%</td>
</tr>
</tbody>
</table>
Conclusion

• Basic Framework: Driver Utility function.
• All “safe” driver state may not be stable.
• Incentive for distracted drivers.
• Impact of liability rule.
• Future research.
  1. Headway as a decision variable (indication of driver aggressiveness).
  2. Estimate of utility function based on observed driver action (use of vehicle trajectory data).
  3. Identify set of optimum liability rules.
THANK YOU!